**Question 1:**

**Consider using a Hierarchical N-gram predictor to predict the next move of your opponent for a fighting game where the only actions are L and R moves. Suppose that we have the following training data (observed sequence of moves):**

**R L R L L R L R R R L R R L L R L R L R L R R L R R L R L R R L L R R L R R L R L L R**

1. **Using a hierarchical 3-gram predictor, what is the predicted next action for input “L R R”, if we want at least 5 samples for prediction?**

3 – GRAM

|  |  |  |  |
| --- | --- | --- | --- |
|  | **L** | **R** | **# of Samples** |
| **LL** | 0/4 | 4/4 | 4 |
| **LR** | 6/14 | 7/14 | 14 |
| **RL** | 4/15 | 11/14 | 15 |
| **RR** | 7/8 | 1/8 | 8 |

Next action is L. Since the input is “L R R”, we first look at the probability of “L R R” and it’s 7/14 = 0.5. We then look for what’s after “R R”, which is the predicted action. “R R L” has a higher probability of 7/8 = 0.875. We realize that 7 out of 9 times after “R R”, we get “L”.

1. **Using a hierarchical 3-gram predictor, what is the predicted next action for input “R R L”, if we want at least 15 samples for prediction?**

3 – GRAM

|  |  |  |  |
| --- | --- | --- | --- |
|  | **L** | **R** | **# of Samples** |
| **LL** | 0/4 | 4/4 | 4 |
| **LR** | 6/14 | 7/14 | 14 |
| **RL** | 4/15 | 11/14 | 15 |
| **RR** | 7/8 | 1/8 | 8 |

Next action is L. Since the input is “R R L”, we first look at the probability of “R R L” and it’s 7/8 = 0.875. We then look for what’s after “R L”, which is the predicted action. “R L R” has a higher probability of 11/14 = 0.786. We realize that 7 out of 9 times after “R L”, we get “R”.

1. **Using a hierarchical 3-gram predictor, what is the predicted next action for input “R L R”, if we want at least 30 samples for prediction?**

3 – GRAM

|  |  |  |  |
| --- | --- | --- | --- |
|  | **L** | **R** | **# of Samples** |
| **LL** | 0/4 | 4/4 | 4 |
| **LR** | 6/14 | 7/14 | 14 |
| **RL** | 4/15 | 11/14 | 15 |
| **RR** | 7/8 | 1/8 | 8 |

Not enough samples for “R L R”. Moving to 2-GRAM:

|  |  |  |  |
| --- | --- | --- | --- |
|  | **L** | **R** | **# of Samples** |
| **L** | 4/19 | 15/19 | 19 |
| **R** | 15/23 | 8/23 | 23 |

Not enough samples for “R L R”. Moving to 1-GRAM:

|  |  |  |  |
| --- | --- | --- | --- |
|  | **L** | **R** | **# of Samples** |
| **-** | 19/43 | 23/43 | 43 |

Next action is R. Since the input is “R R L”, we first look at the probability of “R R L” and it’s 7/8 = 0.875. Since that is below the sample threshold (15), we move to the 2-Gram. We check the probability of what’s after “L”. “L R” has the higher probability of 15/19 = 0.789. Since that’s bellow the threshold as well, we move to the 1-Gram. We check the probability of what’s after “R”. “R” has the higher probability of 23/43, while “L” has 19/43. Therefore, the next action is expected to be “R”.

**Question 2:**

**Consider a sphere with radius 3 and center point (1, 1, 1) colliding with the triangular face of an object. The vertices of the triangle are (0, 5, 0), (4, 0, -1), and (-5, -2, 0). Determine the following:**

1. **the contact normal; and**

Label points:

* c (1, 1, 1)
* P1 (0, 5, 0)
* P2 (4, 0, -1)
* P3 (-5, -2, 0)

**Find collision normal of the plane:**

Taking two vectors formed by the three points and calculating their cross product, you get the contact normal.

We get the cross product:

We invert the vector v to -v and normalize it:

1. **the interpenetration depth.**

The tangent point is found by referencing it to the sphere. We do this by multiplying by the radius to match the sphere’s relative origin and adding (1, 1, 1), since the center of the sphere is located to that point in world space:

We need to find the equation of the plane. -(A, B, C) = normal of plane and (x0, y0, z0) = arbitrary point in the plane, ex: (0, 5, 0):

We normalize the plane to find the distance between the point and the plane. This is done by substituting (x, y, z) we obtained from above in the normalization equation for the plane:

The sphere to find the penetration depth is: 1.5100638